

Collective cavity QED with multiple atomic levels

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We study the transmission spectra of ultracold Rubidium atoms coupled to a high-finesse optical cavity. Under weak probing with π -polarized light, the linear response of the system is that of a collective spin with multiple levels coupled to a single mode of the cavity. By varying the atom number, we change the collective coupling of the system. We observe the change in transmission spectra when going from a regime where the collective coupling is much smaller than the separation of the atomic levels to a regime where both are of comparable size. The observations are in good agreement with a reduced model we developed for our system.

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I. INTRODUCTION

The coherent interaction between light and matter in the context of cavity quantum electrodynamics (cavity QED) has been under intensive study in recent years, especially in the context of quantum information processing [1]. The essence of this interaction is captured by considering a two-level atom coupled to a single mode of the electromagnetic field inside a cavity [2, 3]. However, for many physical systems, two important extensions to this model have to be taken into account. First, real scatterers often have a more complex level structure [4] and second, the coupling for N scatterers coupled to the same cavity mode is collectively enhanced [5]. Both extensions provide useful functionality in the context of quantum information processing. The collective enhancement has recently been used to store multiple microwave pulses in the collective modes of an electron spin ensemble [6], whereas different levels could be used to collectively encode several qubits [7], resulting in quantum repeaters able to perform simple error correction [8].

Studying a system of N alkali-metal atoms coupled to a cavity allows one to study both effects in detail. When probing the system with π -polarized light for example, the transmission spectrum deviates strongly from the simple two-level picture. For a two-level atom, the spectrum shows a single splitting due to an avoided crossing between the energies of the state with one excitation in the atom and the state with one excitation in the cavity [9]. Alkali-metal atoms have multiple hyperfine ground and excited states. In the spectrum, there are avoided crossings associated with the transitions between the different ground and excited states [4] and the net spectrum depends on the relative strength of the transitions and their separation. For N atoms the shape of the spectrum depends on the atom number N as well, because the size of the splittings that correspond to the different avoided

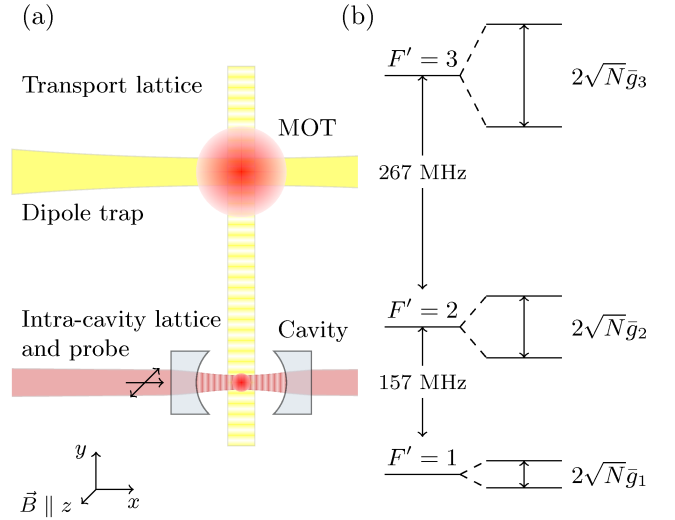


FIG. 1. Setup of the experiment. (a) We trap an unpolarized gas of ultracold ^{87}Rb atoms in a two-dimensional lattice inside the mode of a high-finesse optical cavity and probe the system weakly with π -polarized light. (b) Under weak driving the system behaves as a collective spin with three transitions coupled to a single mode of the cavity. Of interest are the transitions to the excited states $|F' = 1, 2, 3\rangle$ of the $D2$ line. The corresponding splittings are separated by the hyperfine splitting of the excited states. Their size is given by the collective coupling, which in turn depends on the atom number N and an effective single atom coupling constant $\bar{g}_{F'}$. By varying N , we change the collective coupling and are able to explore different regimes of cavity QED with multiple atomic levels.

crossings is collectively enhanced by a factor \sqrt{N} , as has been observed for thermal atoms [10–13] and for Bose-Einstein condensates [14, 15].

Here, we experimentally investigate the transmission spectra of N ^{87}Rb atoms coupled to a high-finesse optical cavity when the system is weakly probed with π -polarized light. By changing the atom number, we are able to go from a regime where the size of the splittings is

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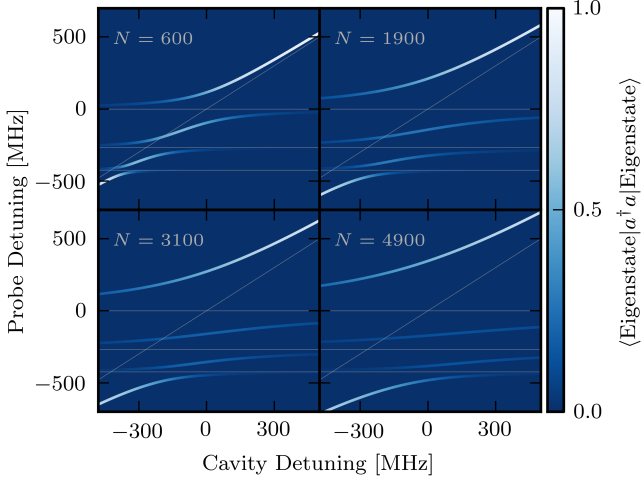


FIG. 2. Theoretical transmission spectra for weak driving. Three avoided crossings between the energy of the bare cavity (solid diagonal line) and the energies of the bare atom (solid horizontal lines) contribute to the transmission spectrum. Transmission through the cavity is expected at the eigenenergies of the system. The amount of transmission is proportional to $\langle a^\dagger a \rangle$ of the corresponding eigenstate, as indicated by color. For low atom numbers the spectrum consists of three separate splittings, while for higher atom numbers these merge into a single splitting. Calculations were performed for an equal distributions of m_F states and a \bar{g} of 6.5 MHz. All detunings are with respect to the $|F = 2\rangle$ to $|F' = 3\rangle$ transition.

much smaller than the separation of the hyperfine excited states to a regime where both are of comparable size. Depending on the regime, we observe markedly different spectra. For low atom numbers, the spectrum shows separate splittings, whereas for higher atom numbers these merge into a single splitting. In the rest of this article, we first extend previous models of cavity QED with multiple hyperfine levels [4, 16–18] to derive an effective Hamiltonian describing our system and then detail the experiments performed and compare them with the predictions from theory.

II. THEORY

Our setup is sketched in Fig. 1 (a). We trap N ^{87}Rb atoms inside the mode of a high-finesse optical cavity. Our cavity is a Fabry-Pérot resonator with no discernible birefringence. Hence, it supports two orthogonal and degenerate modes of the electromagnetic field. By applying a magnetic field perpendicular to the optical axis of the cavity, we ensure that these modes correspond to π and $\perp = (\sigma_+ + \sigma_-)/\sqrt{2}$ transitions of the atom. The magnetic field is calculated to be 2.6 G, based on the coil geometry. We neglect the resulting differential Zeeman shifts of 600 kHz [19] because they are small compared to the

cavity linewidth of 5.3 MHz.

The experiments were performed on the transitions from the $|F = 2\rangle$ ground state to the $|F' = 1, 2, 3\rangle$ excited states of the $D2$ line [19]. A model Hamiltonian describing the system is

$$\begin{aligned} H = & \hbar\omega_C(a^\dagger a + b^\dagger b) + \hbar \sum_{i=1}^N \sum_{F'=1}^3 \omega_{F'} |F'\rangle \langle F'|_i + \\ & \hbar \sum_{i=1}^N \sum_{F'=1}^3 (g_i a^\dagger D_i^0(2, F') + g_i b^\dagger D_i^\perp(2, F') + \text{h.c.}). \end{aligned} \quad (1)$$

Here, a and b are the annihilation operators for the cavity modes corresponding to π and \perp transitions of the atom, ω_C is the resonance frequency of the cavity, $|F'\rangle \langle F'|_i$ is the operator projecting the i -th atom onto the excited state $|F'\rangle$, $\omega_{F'}$ is the frequency of the transition from the $|F = 2\rangle$ ground state to the $|F'\rangle$ excited state, g_i is the complex atom-cavity coupling constant of the i -th atom and h.c. denotes the Hermitian conjugate. $D_i^0(2, F')$ and $D_i^\perp(2, F')$ are the atomic dipole transition operators for the i -th atom from the $|F'\rangle$ excited state to the $|F = 2\rangle$ ground state, which take into account the different coupling strengths for the π and \perp transitions.

We follow the convention in [4] to define the atomic dipole transition operators for an atom interacting with different polarizations of the light field as

$$D^q(F, F') = \sum_{m_F} |F, m_F\rangle \langle F, m_F | \mu_q | F', m_F + q \rangle \langle F', m_F + q |, \quad (2)$$

where $q = \{-1, 0, 1\}$ and μ_q is the dipole operator for $\{\sigma_-, \pi, \sigma_+\}$ -polarization, normalized such that for the cycling transition from the $|F = 2, m_F = 2\rangle$ to the $|F' = 3, m_{F'} = 3\rangle$ state $\langle \mu \rangle = 1$. For \perp -polarized light we identify $D_i^\perp = (D_i^{+1} + D_i^{-1})/\sqrt{2}$.

In the experiment, we probe the system with π -polarized light after having prepared the atoms in a statistical mixture of m_F states. For weak driving and large atom numbers the system behaves like a collective spin with one ground and three excited states coupled to the π -mode of the cavity, as indicated in Fig. 1 (b). We model the system by the effective Hamiltonian

$$\begin{aligned} H = & \hbar\omega_C a^\dagger a + \hbar \sum_{F'=1}^3 \omega_{F'} |F'\rangle \langle F'| + \\ & \hbar \sum_{F'=1}^3 \left(\sqrt{N} \bar{g}_{F'} a^\dagger |F = 2\rangle \langle F'| + \text{h.c.} \right), \end{aligned} \quad (3)$$

which we derive in detail in Appendix A. The coupling of the collective spin is enhanced by a factor \sqrt{N} as compared to an effective single atom coupling constant $\bar{g}_{F'}$ that is an average of both the spatial dependence and

the m_F state dependence of the coupling of the individual atoms. Because we are only considering the case of weak driving, we restrict our discussion of the spectrum to the excitation manifold with a single excitation present in the system [4]. In this manifold, there are three avoided crossings between the energies of the bare cavity and the bare atom. If the crossings are well separated, we expect three splittings in the transmission spectrum of size $2\sqrt{N}\bar{g}_{F'}$, separated by the hyperfine splitting of the excited states of ^{87}Rb . Solving the eigensystem for the Hamiltonian yields a more detailed description of the spectrum. We expect to see transmission when the probe laser frequency is resonant with the eigenenergies of the system, while the amount of transmission is proportional to the expectation value $\langle a^\dagger a \rangle$ of the corresponding eigenstate [4]. Numerical calculations lead us to predict the detailed form of spectrum as summarized in Fig. 2.

III. EXPERIMENT

We verify our predictions using the experimental setup sketched in Fig. 1 (a). Its central element is a cavity made of two coned down spherical mirrors with radii of curvature of 25 mm separated by 500 μm . The transmission per mirror is ≈ 20 ppm and the losses are ≈ 35 ppm, as estimated from measurements of the transmission, reflection and linewidth of the cavity. In terms of cavity QED parameters, our setup is described by $(g, \kappa, \gamma) = 2\pi \times (9.2, 2.6, 3.0)$ MHz, where g is the maximum single atom coupling constant for the $|F=2, m_F=2\rangle$ to $|F'=3, m_F=3\rangle$ transition, κ is the cavity field decay rate, and γ is the atomic dipole decay rate [19, 20].

We start the experiment by loading atoms into an optical dipole trap in the $|F=1\rangle$ ground state, similar to our previous experiments [21]. The dipole trap is formed by a beam with a wavelength of 1064 nm and a power of 12 W focused to a waist of 25 μm . From there the atoms are evaporatively cooled into a transfer lattice formed by two counter-propagating beams of the same wavelength. Each lattice beam has a maximum power of 1 W and is focused to a waist of 50 μm inside the cavity. By changing their relative frequency, the atoms are transported into the cavity [22], which is located 9.2 mm below the position of the dipole trap. Inside the cavity, the atoms are trapped in a two-dimensional lattice formed by the transfer lattice and an intra-cavity beam at 808 nm with a circulating power of 3 mW and a waist of 25 μm . We also use this beam to stabilize the length of the cavity.

Within the cavity, the atoms are repumped into the $|F=2\rangle$ ground state and the repumping beam is left on during the rest of the experiment. The atom-cavity system is probed through the cavity with light linearly polarized along the axis of quantization defined by the magnetic field. The intensity of the probe is adjusted to give a small mean photon number \bar{n}_{empty} in the empty cavity. It is chosen to be as close as possible to the weak probing condition $\bar{n}_{\text{empty}} \ll 1$ while still resulting in enough

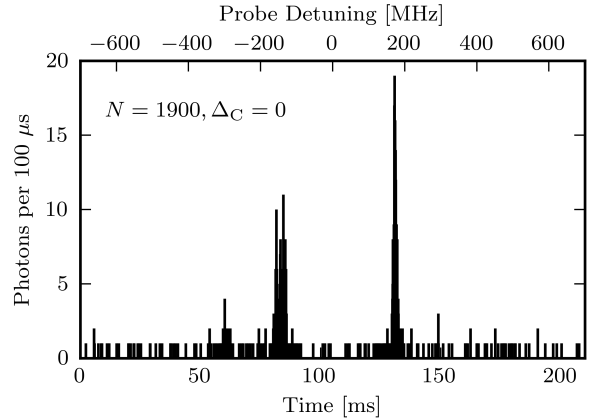


FIG. 3. Cavity transmission. For a given experiment we fix the cavity detuning Δ_C and the atom number N . The probe laser detuning is swept from -700 MHz to $+700$ MHz in 210 ms, which is small compared to the measured atom number lifetime of 3 s. At the end of the sweep the atom number is measured by absorption imaging. The cavity output is directed onto a single photon counting module and the counts are binned in 100 μs intervals. All detunings are with respect to the $|F=2\rangle$ to $|F'=3\rangle$ transition.

signal for the experiment. The transmission spectrum is recorded by directing the output of the cavity onto a single photon counting module. After probing the system, we measure the atom number via absorption imaging.

For each experiment the cavity detuning $\Delta_C = \omega_C - \omega_1$ is fixed and we control the number of atoms by adjusting the power in the initial dipole trap. During the experiment the probe laser frequency is swept over the frequency range of interest, resulting in a transmission signal as shown in Fig. 3. By repeating the experiment for different cavity detunings and different average atom numbers, we map out the transmission spectra, as shown in Fig. 4.

IV. CONCLUSION

To compare the measured spectra with our model predictions, we assume equal population of both the m_F states and the lattice sites of the intra-cavity trap. For simplicity, we neglect the finite size of the cloud along the direction transverse to the optical axis of the cavity [23]. Because the intra-cavity trap has a different lattice spacing than the mode of the cavity, the spatial averaging of g_i gives $\bar{g} \approx \max(g_i)/\sqrt{2} = 6.5$ MHz, as for a uniform distribution of atoms along the cavity axis. The agreement between the experiment and the model, as illustrated in Fig. 2 and Fig. 4, is obtained without any free parameters.

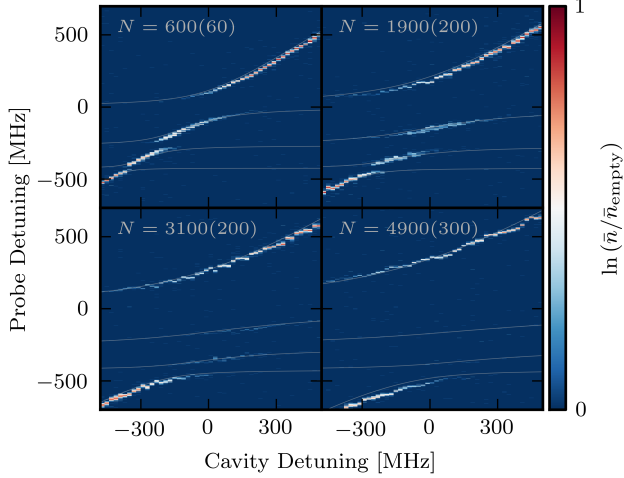


FIG. 4. Experimental transmission spectra. By varying the cavity detuning we map out the full transmission spectra of the system for different average atom numbers N , with their standard deviation indicated in parentheses. The recorded transmission is indicated in color in terms of intra-cavity photon number \bar{n} , normalized to the average photon number inside the empty cavity on resonance \bar{n}_{empty} , where $\bar{n}_{\text{empty}} = 0.25$ for the top left and bottom right plots and $\bar{n}_{\text{empty}} = 0.35$ for the top right and bottom left plots. Points with counts below 2 are colored as 0. The gray solid lines indicate the positions of the theoretical eigenenergies for the measured average atom number N , an equal distributions of m_F states and a \bar{g} of 6.5 MHz. All detunings are with respect to the $|F = 2\rangle$ to $|F' = 3\rangle$ transition.

So far, we have neglected the effect of non-negligible driving strength. Our model assumes weak driving in order to describe the system as a collective spin. It will fail for higher driving strength since it predicts a strong non-linearity due to the saturation of the single spin. However, as the atom number increases, the system saturates at higher excitation numbers and loses the strong non-linearity predicted by our simple model. Another effect we have neglected are the m_F state changing processes either due to optical pumping or due to the weak relaxation via the undriven mode of the cavity. This is justified because the experiment was performed with a large number of atoms in a statistical mixture of all m_F states. Thus, residual m_F state changing processes are not expected to alter the distribution significantly. In a series of related experiments, we have started to investigate the dynamics of polarized gases, where the atoms are optically pumped into a particular m_F state prior to their interaction with the cavity. In this case, we see evidence of m_F changing processes, which in turn change the effective coupling $\bar{g}_{F'}$ of the system. These processes lead to complex dynamics, which will be an interesting topic for future investigations.

In summary, we have experimentally demonstrated that the linear response of a gas of alkali-metal atoms

coupled to a high-finesse optical cavity is well described by a collective spin with multiple levels coupled to a single mode of the cavity. Using this system provides a flexible testbed for collective cavity QED with multiple atomic levels and the theoretical frame work is applicable to other physical systems used in quantum information processing. These include hybrid systems like alkali-metal atoms above a coplanar waveguide, collectively coupled to linearly polarized microwave photons [24], and nitrogen-vacancy centers coupled to a superconducting resonator, where a similar effect in the transmission spectrum has recently been observed [25].

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Appendix A: Derivation of the reduced Hamiltonian

In order to derive the effective Hamiltonian of Eq. 3, we take a closer look at the interaction part H_I of the full Hamiltonian of Eq. 1, that is,

$$H_I = \sum_{i=1}^N \sum_{F'=1}^3 (g_i a^\dagger D_i^0(2, F') + g_i b^\dagger D_i^\perp(2, F') + \text{h.c.}). \quad (\text{A1})$$

We will first consider the driven π -mode and then the undriven \perp -mode. Driving the π -mode of the cavity populates the state where all the atoms are in the ground state and there is one π -photon in the cavity,

$$|\pi, g_N\rangle = |\pi\rangle \prod_{i=1}^N |F, m_F\rangle_i. \quad (\text{A2})$$

Subsequent interaction with the cavity leads to transitions to the Dicke state, where one of the atoms is excited and the cavity is empty,

$$|0, e_N\rangle = |0\rangle \left(\frac{1}{\sqrt{N}} \sum_{i=1}^N |F', m_{F'}\rangle_i \prod_{j \neq i} |F, m_F\rangle_j \right). \quad (\text{A3})$$

The rate of which is collectively enhanced, i.e.,

$$\langle 0, e_N | H_I | \pi, g_N \rangle = \sqrt{N} \bar{g}_{F'}. \quad (\text{A4})$$

To calculate the effective single atom coupling rate $\bar{g}_{F'}$, we note that the coupling of the atom to the π -mode of the cavity depends on its position relative to the cavity field. Along the cavity axis the corresponding g_i varies from maximum coupling for an atom at an anti-node of

the cavity field, to zero coupling for an atom at a node. In the transverse direction the coupling varies as the Gaussian field distribution of the TEM₀₀ mode of the Fabry-Pérot cavity.

In addition to position, the coupling depends on the m_F state of the atom via the Clebsch-Gordon coefficient $\langle F, m_F | \mu_0 | F', m_F \rangle_i$ that enters the dipole transition operator

$$D_i^0(F, F') = \sum_{m_F} |F, m_F\rangle_i \langle F, m_F | \mu_0 | F', m_F \rangle_i \langle F', m_F |_i. \quad (\text{A5})$$

For a product state of N atoms, each in a particular m_F state, the sum in the dipole transition operator $D_i^0(2, F')_i$ has only one non-zero term for every atom. The operator then becomes a lowering operator, taking the i -th atom from the $|F'\rangle$ to the $|F=2\rangle$ state, with a prefactor given by the Clebsch-Gordon coefficient and the spatially varying coupling constant g_i , i.e.,

$$g_i D_i^0(2, F')_i = g_i \langle F, m_F | \mu_0 | F', m_F \rangle_i |F=2\rangle \langle F'|_i. \quad (\text{A6})$$

For weak driving the ensemble behaves as a collective spin and the effective single atom coupling constant results from both an average over the spatial distribution of the atoms and their m_F states, that is

$$\bar{g}_{F'} = \sqrt{\frac{1}{N} \sum_{i=1}^N |\langle F, m_F | \mu_0 | F', m_F \rangle_i g_i|^2} \quad (\text{A7})$$

$$\approx \bar{g} \sqrt{\sum_{m_F} p(m_F) |\langle F, m_F | \mu_0 | F', m_F \rangle|^2}. \quad (\text{A8})$$

Here, $\bar{g} = \sqrt{\sum_i |g_i|^2 / N}$ is the coupling constant resulting from spatial variations in g_i and $p(m_F)$ is the relative

population in the different m_F states. In the last step, we have assumed that enough atoms are in all of the m_F states such that the atoms in each state independently average to \bar{g} .

The undriven mode of the cavity is only populated by a transition of the excited state $|0, e_N\rangle$ into the state where the cavity holds a \perp -photon and the atoms are in a superposition of one of them having changed m_F states, i.e.,

$$|\perp, g'_N\rangle = |\perp\rangle \left(\frac{1}{\sqrt{2N}} \sum_{q=\pm 1} |F, m_F + q\rangle_i \prod_{j \neq i} |F, m_F\rangle_j \right). \quad (\text{A9})$$

The rate at which this process occurs is not collectively enhanced [16, 18], i.e.,

$$\langle \perp, g'_N | H_I | 0, e_N \rangle = \bar{g}_{F'}, \quad (\text{A10})$$

even if all m_F states are macroscopically occupied [17]. Because $N \gg 1$ in the experiment, we restrict our treatment to the interaction with the driven π -mode of the cavity and arrive at the effective Hamiltonian for weak driving

$$H = \hbar \omega_C a^\dagger a + \hbar \sum_{F'=1}^3 \omega_{F'} |F'\rangle \langle F'| + \hbar \sum_{F'=1}^3 \left(\sqrt{N} \bar{g}_{F'} a^\dagger |F=2\rangle \langle F'| + \text{h.c.} \right). \quad (\text{A11})$$

The treatment above is similar to work in the area of quantum dots studying the vacuum-Rabi splitting in the presence of inhomogeneous broadening [26].

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